Exams 2024

Model (1)

15 Marks

1 Choose the correct answer: 3

a) If $2^6 \times 5^6 = 10^m$, then m = ...

1) 5 2) 12 3) 6 4) 36

b) The domain of the function f: f $(x) = \dfrac{x^2}{x^3 - x}$ is ....

1) {0 , 1} 2) {0 , 1 , −1} 3) {1 , −1} 4) {1}

c) The probability of the sure event is ....

1) 0 2) 1 3) 2 4) −1

d) If $A \subset S$ for any random experiment and $P(A) = \frac{2}{3}$, then $P'(A)$= ....

1) $\frac{2}{3}$ 2) 1 3) $\frac{1}{3}$ 4) $\frac{3}{2}$

e) If there are infinite numbers of solutions to the two equations:

$x + 4y = 7$, $3x + ky = 21$ in $R \times R$, then k = ...

1) 12 2) 8 3) 4 4) 3

f) If $6x - 7y = 0$, then $\frac{x}{y}$ =.......................... .

1) $\frac{6}{7}$ 2) $\frac{7}{6}$ 3) $-\frac{6}{7}$ 4) $-\frac{7}{6}$

2 a) Find the solution set of the equation: $x^2 - 4x + 2 = 0$ in R using the general formula. 3

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b) Two positive real numbers, the difference between them is 1 and the sum of their squares is 25, find the two numbers.

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3

3 **a** Find n(x) in the simplest form, showing the domain:

$$n(x) = \frac{2x^2 - x - 6}{x^2 - 3x} \div \frac{4x^2 - 9}{2x^2 - 3x}$$

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b If $n(x) = \dfrac{x^2 - 2x}{x - 2}$

1st Find $n^{-1}(x)$ in the simplest form.

2nd If $n^{-1}(x) = \frac{1}{3}$, find the value of x.

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4 **a** If $n_1(x) = \dfrac{x^2}{x^3 - x^2}$, $n_2(x) = n_2(x) = \dfrac{x^3 + x^2 + x}{x^4 - x}$ Prove that $n_1 = n_2$

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b Find n(x) in the simplest form, showing the domain:

$$n(x)=\frac{x^2-2x}{x^2-3x+2}-\frac{x^2-4}{x^2+x-2}$$

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5 **a** If A, B are two events of the sample space of random experiment:

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P (A) = $\frac{1}{3}$, P(A∪B) = $\frac{7}{12}$, P(B) = x, find the value of x if:

1 A and B are mutually exclusive events. **2** A⊂B

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b Find in R x R, the S.S. of the following equations:

$2x - y = 3$ (1) , $x + 2y = 4$ (2)

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15 Marks

Model (2)

1 Choose the correct answer:

3

a If $x^2 - y^2 = 45$, $x - y = 5$, then $x + y =$ ....

1 8 2 5 3 9 4 6

b If A and B are mutually exclusive events, then $P(A \cap B) =$ ....

1 1 2 0 3 $\varnothing$ 4 $\frac{1}{2}$

c If {−2 , 2} is the set of zeros of the function f where $f(x) = x^2 + a$, then a = ...

1 −4 2 4 3 2 4 −2

d If $4^n = 3$, the n $64^n =$ ....

1 9 2 48 3 27 4 3

e The intersection point of the two straight lines: $x - 1 = 0$, $y = 2$ is ....

1 (1 , 2) 2 (−2 , 1) 3 (−1 , 2) 4 (−1 , −2)

f $x^0 = 1$ If $x \in R - \{$ .... $\}$

1 2 2 0 3 1 4 −1

2

3

a If $n(x) = \frac{x - 3}{x^2 - 6x + 9} + \frac{x^2 + x3 + 9}{x^3 - 27}$

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b Find the solution set in R x R for two equations:

$x - y = 4$, $x^2 - 3y^2 = 22$

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3

3 **a** Find n(x) in the simplest form, showing the domain of n where:

$$n(x)=\frac{x^3-1}{x^2+4x-5}\times\frac{x+5}{x^2+x+1}$$

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b If A and B are two events from the sample space of a random experiment and

P(A) = 0.4, P(B)= 0.5 and P(A∩B)= 0.2

1 P(A′) **2** P(A∪B) **3** P(A–B)

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3

4 **a** By using the general formula, find the solution set of the following equation in R:

$x^2+3x-3=0$ (Approximating the result to the nearest one decimal place)

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b Find the S.S. of the following equations in R:

$x-3y=5$, $3x+2y=4$

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5 **a** If $n(x) = \frac{x^2 - 3x - 4}{x^2 - 16}$:

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1. Find $n^{-1}(x)$

2. If $n^{-1}(x) = \frac{1}{2}$, find the value of x.

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b Find $n(x)$ in the simplest form, showing the domain of n:

$$n(x) = \frac{x^3 - 8}{x^2 - 2x - 15} \div \frac{x^2 + 2x + 4}{x - 5}$$

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15 Marks

Model (3)

1 Choose the correct answer:

3

a If $2^x = 5$, $5^y = 20$, then $\frac{xy}{x+2} =$ ....

1. $\frac{1}{4}$ 2. $\frac{1}{2}$ 3. 1 4. 2

b If $n(x)= \frac{x^2-3x}{(x-3)(x^2+4)}$ then the domain of n^{-1} is ....

1. $R-\{0,-4\}$ 2. $R-\{0,3\}$ 3. $R-\{0\}$ 4. R

c If $A \subset S$ in a random experiment and $P(A')=3P(A)$, then $P(A)=$ ....

1. $\frac{1}{3}$ 2. $\frac{1}{2}$ 3. $\frac{1}{5}$ 4. $\frac{1}{4}$

d The set of zeros of the function $n(x)= x^2+25$ is ....

1. $\{5\}$ 2. $\{\pm5\}$ 3. $\varnothing$ 4. 0

e If the area of a square is 72 cm^2, then its diagonal length of the square = ...cm

1. 6 2. $6\sqrt{2}$ 3. 12 4. 18

f If the S.S. of the equation: $x^2 - ax + 4 = 0$ is $\{-2\}$, then a = ...

1. -2 2. -4 3. 2 4. 4

2

3

a Find n(x) in the simplest form, showing the domain:

$$n(x) = \frac{x-3}{x^2-7x+12} - \frac{x-3}{3-x}$$

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b Find the solution set in R × R for the two equations:

$x + y = 5$, $x^2 + xy = 15$

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3 **a** Find the S.S. of the equation: $(x-2)(x+4)+3=0$, using the general formula. (Rounding the results to two decimal places): 3

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b If $n_1(x)=\frac{2x}{2x+4}$, $n_2(x)=\frac{x^2+2x}{x^2+4x+4}$, prove that $n_1=n_2$.

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4 **a** If A and B are two events in a random experiment and 3

$P(A)=P(A')$, $P(A \cap B)=\frac{1}{16}$, $P(B)=\frac{5}{8}P(A)$

Find: **1** P(B)

2 $P(A \cup B)$

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b Find $n(x)$ in the simplest form, showing the domain of n where:

$n(x)=\frac{x^3-1}{x^2-2x+1} \times \frac{2x-2}{x^2+x+1}$

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5 **a** Find the set of solution of the two following equations in R x R: 3

$x-y=4$ $2x+y=5$

....

....

....

b A rectangle with a length more than its width by 4 cm, if the perimeter of the rectangle is 28 cm, find the area of the rectangle.

....

....

....

15 Marks

Model (1)

1 Choose the correct answer:

3

a If $2^6 \times 5^6 = 10^m$, then m = ...

1 5 2 12 3 6 4 36

b The domain of the function f: f $(x) = \dfrac{x^2}{x^3 - x}$ is ....

1 {0 , 1} 2 {0 , 1 , –1} 3 {1 , –1} 4 {1}

c The probability of the sure event is ....

1 0 2 1 3 2 4 –1

d If A⊂S for any random experiment and $P(A) = \frac{2}{3}$, then P´(A)= ....

1 $\frac{2}{3}$ 2 1 3 $\frac{1}{3}$ 4 $\frac{3}{2}$

e If there are infinite numbers of solutions to the two equations:

$x + 4y = 7$, $3x + ky = 21$ in R × R, then k = ...

1 12 2 8 3 4 4 3

f If $6x - 7y = 0$, then $\frac{x}{y}$ = ...

1 $\frac{6}{7}$ 2 $\frac{7}{6}$ 3 $-\frac{6}{7}$ 4 $-\frac{7}{6}$

2 a Find the solution set of the equation: $x^2 - 4x + 2 = 0$ in R using the general formula.

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a = 1 b = –4 c=2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 2}}{2 \times 1} = \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$$

Then $x = 2 + \sqrt{2}$ or $x = 2 - \sqrt{2}$

S.S. $= \{2 + \sqrt{2}, 2 - \sqrt{2}\}$

b Two positive real numbers, the difference between them is 1 and the sum of their squares is 25, find the two numbers.

Let the numbers be x and y.

$x - y = 1$ (1) $x^2 + y^2 = 25$ (2)

$x = y + 1$ (3)

From (3) in (2)

$(y + 1)^2 + y^2 = 25$

$y^2 + 2y + 1 + y^2 - 25 = 0$

$2y^2 + 2y - 24 = 0$

$2(y^2 + y - 12) = 0$

$2(y + 4)(y - 3) = 0$

$y = -4$ or $y = 3$

Rejected $x = 3 + 1$

$x = 4$ the two numbers are 3 and 4.

3 **a** Find $n(x)$ in the simplest form, showing the domain: 3

$$n(x) = \frac{2x^2 - x - 6}{x^2 - 3x} \div \frac{4x^2 - 9}{2x^2 - 3x}$$

$$n(x) = \frac{(2x + 3)(x - 2)}{x(x - 3)} \div \frac{(2x + 3)(2x - 3)}{x(2x - 3)}$$

The domain of $n(x) = R - \{0 , 3 , \frac{3}{2} , -\frac{3}{2}\}$

$$n(x) = \frac{x - 2}{x(x - 3)}$$

b If $n(x) = \dfrac{x^2 - 2x}{x - 2}$

1st Find $n^{-1}(x)$ in the simplest form.

2nd If $n^{-1}(x) = \frac{1}{3}$, find the value of x.

1st $n(x) = \dfrac{x(x - 2)}{(x - 2)}$, $n^{-1}(x) = \dfrac{x - 2}{x(x - 2)}$

The domain of $n^{-1} = R - \{0 , 2\}$

$$n^{-1}(x) = \frac{1}{x}$$

2nd $n^{-1}(x) = \frac{1}{3}$, $x = 3$

4 **a** If $n_1(x) = \dfrac{x^2}{x^3 - x^2}$, $n_2(x) = n_2(x) = \dfrac{x^3 + x^2 + x}{x^4 - x}$ Prove that $n_1 = n_2$ 3

$n_1(x) = \dfrac{x^2}{x^2(x - 1)}$

The domain of n1 $= R - \{0 , 1\}$

$n_1(x) = \dfrac{1}{x - 1}$

$n_2(x) = \dfrac{x(x^2 + x + 1)}{x(x - 1)(x^2 + x + 1)}$

the domain of n2 $= R - \{0 , 1\}$

$n_2(x) = \dfrac{1}{x - 1}$

$\therefore n_1(x) = n_2(x)$, domain of n_1 = domain of n_2

$\therefore n_1 = n_2$

b Find n(x) in the simplest form, showing the domain:

$$n(x)=\frac{x^2-2x}{x^2-3x+2}-\frac{x^2-4}{x^2+x-2}$$

$$n(x)=\frac{x(x-2)}{(x-2)(x-1)}-\frac{(x-2)(x+2)}{(x+2)(x-1)}$$

$\therefore$ The domain of n = R – {2 , 1 , –2}

$$n(x)=\frac{x}{(x-1)}-\frac{x-2}{(x-1)}=\frac{2}{(x-1)}$$

5 **a** If A, B are two events of the sample space of random experiment: 3

P (A) = $\frac{1}{3}$, P(A∪B) = $\frac{7}{12}$, P(B) = x, find the value of x if:

1 A and B are mutually exclusive events. **2** A⊂B

1 $\because$ A and B are mutually exclusive events

$\therefore$ P(A∩B) = 0

$\therefore$ P(A∪B) = P (A) + P(B)

$\frac{7}{12}=\frac{1}{3}+x$ $\therefore x=\frac{7}{12}-\frac{1}{3}=\frac{1}{4}$

2 $\because$ A ⊂ B $\therefore$ P(A∩B) = P (A)

$\therefore$ P(A∪B) = P (A) + P(B) – P(A∩B)

$\therefore \frac{7}{12}=\frac{1}{3}+x-\frac{1}{3}$

$\therefore x=\frac{7}{12}$

Other solution (2): $\because$ A ⊂ B $\therefore$ P(B) = P(A∪B) $\therefore x=\frac{7}{12}$

b Find in R x R, the S.S. of the following equations:

$2x - y = 3$ (1) , $x + 2y = 4$ (2)

$2x - y = 3$ multiply × 2

$x + 2y = 4$

$4x - 2y = 6$

$x + 2y = 4$

-------------- (by adding)

$\therefore 5x = 10$

$\therefore x = 2$ in (1)

$2(2) - y = 3$,

$4 - y = 3$

$\therefore$ y = 1 and S.S. = {(2 , 1)}

15 Marks

Model (2)

1 Choose the correct answer:

3

a If $x^2 - y^2 = 45$, $x - y = 5$, then $x + y =$ ....

1 8   2 5   3 9   4 6

b If A and B are mutually exclusive events, then $P(A \cap B) =$ ....

1 1   2 0   3 Ø   4 $\frac{1}{2}$

c If {–2 , 2} is the set of zeros of the function f where $f(x) = x^2 + a$, then a = ...

1 –4   2 4   3 2   4 –2

d If $4^n = 3$, the n $64^n =$ ....

1 9   2 48   3 27   4 3

e The intersection point of the two straight lines: $x - 1 = 0$, $y = 2$ is ....

1 (1 , 2)   2 (–2 , 1)   3 (–1 , 2)   4 (–1 , –2)

f $x^0 = 1$ If $x \in$ R – { .... }

1 2   2 0   3 1   4 –1

2

3

a If $n(x) = \frac{x-3}{x^2-6x+9} + \frac{x^2+x3+9}{x^3-27}$

$n(x) = \frac{x-3}{(x-3)(x-3)} + \frac{x^2+3x+9}{(x-3)(x^2+3x+9)}$

the domain of n =R –{3}

$n(x) = \frac{1}{(x-3)} + \frac{1}{(x-3)} = \frac{2}{(x-3)}$

b Find the solution set in R x R for two equations:

$x - y = 4$, $x^2 - 3y^2 = 22$

$x - y = 4$ (1) $x^2 - 3y^2 = 22$ (2)

$x = y + 4$ (3)

From 3 in 2:

$(y + 4)^2 - 3y^2 = 22$

$y^2 + 8y + 16 - 3y^2 = 22$

$-2y^2 + 8y + 16 - 22 = 0$

$2y^2 - 8y + 6 = 0$

$(2y-2)(y-3)=0$

$\therefore 2y=2$ or $y=3$

$y=1$

$x=4+1=5$ | $x=3+4=7$

S.S. $=\{(5,1),(7,3)\}$

3 **a** Find $n(x)$ in the simplest form, showing the domain of n where:

3

$$n(x)=\frac{x^3-1}{x^2+4x-5}\times\frac{x+5}{x^2+x+1}$$

$$n(x)=\frac{(x-1)(x^2+x+1)}{(x+5)(x-1)}\times\frac{x+5}{x^2+x+1}$$

$\therefore$ The domain of n $= R-\{-5,1\}$

$n(x)=1$

b If A and B are two events from the sample space of a random experiment and

$P(A)=0.4$, $P(B)=0.5$ and $P(A\cap B)=0.2$

1. $P(A')$
2. $P(A\cup B)$
3. $P(A-B)$

1. $P(A')=1-P(A)=1-0.4=0.6$
2. $P(A\cup B)=P(A)+P(B)-P(A\cap B)=0.4+0.5-0.2=0.7$
3. $P(A-B)=P(A)-P(A\cap B)=0.4-0.2=0.2$

4 **a** By using the general formula, find the solution set of the following equation in R:

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$x^2+3x-3=0$ (Approximating the result to the nearest one decimal place)

$a=1$, $b=3$, $c=-3$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-3\pm\sqrt{3^2-4\times1\times-3}}{2\times1}=\frac{-3\pm\sqrt{21}}{2}$$

$\therefore x\simeq 0.8$ or $x\simeq -3.8$

S.S. $=\{0.8,-3.8\}$

b Find the S.S. of the following equations in R:

$x-3y=5$, $3x+2y=4$

$x-3y=5$ (1) $3x+2y=4$ (2)

$x=5+3y$ (3)

From (3) in (2)

$3(5+3y)+2y=4$

$15 + 9y + 2y = 4$

$11y = -11$

$Y = -1$

By substituting in 3

$x = 5 + 3(-1) = 2$

5 a If $n(x) = \frac{x^2 - 3x - 4}{x^2 - 16}$:

3

1 Find $n^{-1}(x)$

2 If $n^{-1}(x) = \frac{1}{2}$, find the value of x.

1 $n(x) = \frac{(x-2)(x+1)}{(x+4)(x-4)}$ $\qquad n^{-1}(x) = \frac{(x+4)(x-4)}{(x-4)(x+1)}$

the domain of $n^{-1} = R - \{4, -1, -4\}$

$n^{-1}(x) = \frac{(x+4)}{(x+1)}$

2 $\because n^{-1}(x) = \frac{1}{2}$

$\therefore \frac{(x+4)}{(x+1)} = \frac{1}{2}$

$\therefore 2x + 8 = x + 1$

$\therefore x = -7$

b Find $n(x)$ in the simplest form, showing the domain of n:

$n(x) = \frac{x^3 - 8}{x^2 - 2x - 15} \div \frac{x^2 + 2x + 4}{x - 5}$

$n(x) = \frac{(x-2)(x^2 + 2x + 4)}{(x-5)(x+3)} \div \frac{x^2 + 2x + 4}{x - 5}$

The domain of $n = R - \{5, -3\}$

$n(x) = \frac{(x-2)(x^2 + 2x + 4)}{(x-5)(x+3)} \times \frac{x - 5}{x^2 + 2x + 4} = \frac{(x-2)}{(x+3)}$

15 Marks

Model (3)

3

1 Choose the correct answer:

a If $2^x = 5$, $5^y = 20$, then $\frac{xy}{x+2} =$ ....

1 $\frac{1}{4}$ 2 $\frac{1}{2}$ 3 1 4 2

b If $n(x) = \frac{x^2 - 3x}{(x-3)(x^2+4)}$ then the domain of n^{-1} is ....

1 $R - \{0, -4\}$ 2 $R - \{0, 3\}$ 3 $R - \{0\}$ 4 R

c If $A \subset S$ in a random experiment and $P(A') = 3P(A)$, then $P(A) =$ ....

1 $\frac{1}{3}$ 2 $\frac{1}{2}$ 3 $\frac{1}{5}$ 4 $\frac{1}{4}$

d The set of zeros of the function $n(x) = x^2 + 25$ is ....

1 $\{5\}$ 2 $\{\pm 5\}$ 3 $\varnothing$ 4 0

e If the area of a square is 72 cm^2, then its diagonal length of the square = ...cm

1 6 2 $6\sqrt{2}$ 3 12 4 18

f If the S.S. of the equation: $x^2 - ax + 4 = 0$ is $\{-2\}$, then a = ...

1 -2 2 -4 3 2 4 4

2

3

a Find $n(x)$ in the simplest form, showing the domain:

$$n(x) = \frac{x-3}{x^2 - 7x + 12} - \frac{x-3}{3-x}$$

$$n(x) = \frac{x-3}{(x-4)(x-3)} - \frac{x-3}{-(x-3)}$$

The domain n $= R - \{4, 3\}$

$$n(x) = \frac{1}{(x-4)} + 1 = \frac{1}{(x-4)} + \frac{(x-4)}{(x-4)} = \frac{(x-3)}{(x-4)}$$

b Find the solution set in $R \times R$ for the two equations:

$x + y = 5$, $x^2 + xy = 15$

$x + y = 5$ (1) $x^2 + xy = 15$ (2)

$y = 5 - x$ (3)

From (3) in (1):

$x^2 + x(5 - x) = 15$

$x^2 + 5x - x^2 = 15$

$5x = 15 \qquad , x = 3$

in 3

$y = 5 - 3 = 2 \qquad$, S.S. = {3 , 2}

3 **a** Find the S.S. of the equation: $(x-2)(x+4)+3 = 0$, using the general formula. (Rounding the results to two decimal places): **3**

$x^2 + 2x - 8 + 3 = 0$

$x^2 + 2x - 5 = 0$

$a = 1 \qquad , b = 2 \qquad , c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times -5}}{2 \times 1}$$

$$= \frac{-2 \pm \sqrt{4 + 20}}{2} = \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 \pm 2\sqrt{6}}{2}$$

then $x = \frac{-2 + 2\sqrt{6}}{2}$ or $x = \frac{-2 - 2\sqrt{6}}{2}$

$x \simeq 1.45$ or $x \simeq -3.45$

S.S. = {1.45 , −3.45 }

b If $n_1(x) = \frac{2x}{2x+4}$, $n_2(x) = \frac{x^2 + 2x}{x^2 + 4x + 4}$, prove that $n_1 = n_2$.

$n_1(x) = \frac{2x}{2x+4}$

The domain of $n_1 = R - \{-2\}$

$n_1(x) = \frac{x}{x+2}$

$n_2(x) = \frac{x(x+2)}{(x+2)^2}$

The domain of $n_2 = R - \{-2\}$

$n_2(x) = \frac{x}{x+2}$

$\because$ The domain of n_1 = the domain of n_2 , $n_1(x) = n_2(x)$

$\therefore n_1 = n_2$

4 **a** If A and B are two events in a random experiment and **3**

$P(A) = P(A')$, $P(A \cap B) = \frac{1}{16}$, $P(B) = \frac{5}{8} P(A)$

Find: 1 P(B)

2 $P(A \cup B)$

$\because P(A) = P(A')$ $\qquad \therefore P(A) = \frac{1}{2}$

$\therefore P(B) = \frac{5}{8} P(A)$ $\qquad \therefore P(B) = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{5}{16} - \frac{1}{16} = \frac{12}{16} = \frac{3}{4}$$

b Find $n(x)$ in the simplest form, showing the domain of n where:

$$n(x)=\frac{x^3-1}{x^2-2x+1}\times\frac{2x-2}{x^2+x+1}$$

$$n(x)=\frac{(x-1)(x^2+x+1)}{(x-1)(x-1)}\times\frac{2(x-1)}{x^2+x+1}$$

The domain of n = R –{1} , $n(x) = 2$

5 **a** Find the set of solution of the two following equations in R x R:

3

$x - y = 4$ $2x + y = 5$

$x - y = 4$ (1)

$2x + y = 5$ (2) by adding

$3x = 9$, $x = 3$

in (1)

$3 - y = 4$

$-y = 1$

$y = -1$

$\therefore$ S.S. = {(3 , –1)}

b A rectangle with a length more than its width by 4 cm, if the perimeter of the rectangle is 28 cm, find the area of the rectangle.

Let the length = x , the width = y

$x - y = 4$ (1)

$x + y = 14$ (2)

$2x = 18$

$x = 9$

in (1)

$y = 5$

The length = 9 cm and the width = 5 cm

The area of the rectangle = $9 \times 5 = 45$ cm^2

15 Marks — Model (1)

1 Choose the correct answer:

3

a The sum of measures of angles in the circle with radius r is ....

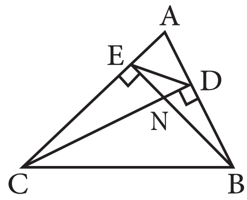
1 πr 2 270° 3 $2\pi r$ 4 360

b The center of the inscribed circle of a triangle is the intersection point of its ....

1 medians 2 axes of symmetry of its sides
3 altitudes 4 bisectors of its interior angles

c In the opposite figure:

How many cyclic quadrilaterals are there? ....



1 4 2 2
3 1 4 3

d The diameter length of a circle is 8 cm. If the straight line L is at a distance of 4 cm from the center, then the straight line L is ....

1 a secant to the circle 2 an axis of symmetry to the circle
3 a tangent to the circle 4 outside the circle

e The measure of the exterior angle at any vertex of a cyclic quadrilateral .... the measure of the interior angle at the opposite vertex.

2 $>$ 2 $<$ 3 $=$ 4 $\geq$

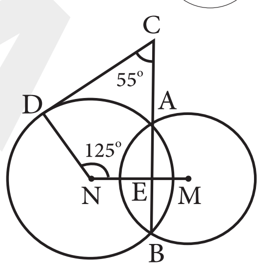
2 In the opposite figure:

3

M and N are two intersecting circles at A and B,

$C \in \overline{BA}$, $D \in$ the circle N, m ($\angle MND$) = 125° and m ($\angle BCD$) = 55°,

Prove that: $\overleftrightarrow{CD}$ is a tangent to the circle N at D.



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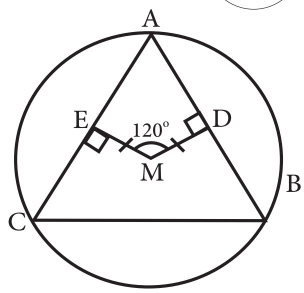
3 **In the opposite figure:**

A circle M, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$, where MD = ME,

m($\angle$DME) = 120°, prove that: the triangle ABC is equilateral.



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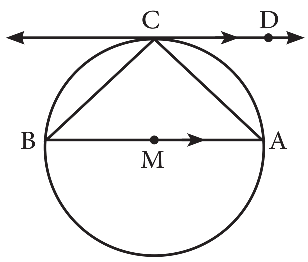
4 **In the opposite figure:**

$\overline{CD}$ is a tangent to the circle M at C, $\overrightarrow{CB} \mathbin{/\!/} \overline{BA}$

Prove that: m ($\angle$DCA) = 45°



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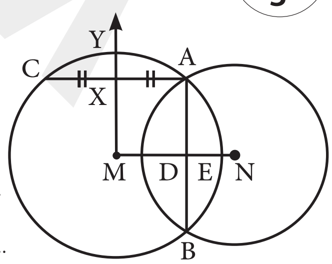
5 **In the opposite figure:**

If M and N are two intersecting circles at A and B,

AB = AC, X is the midpoint of $\overline{AC}$.

Prove that: XY = DE



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15 Marks

Model (2)

1 Choose the correct answer:

3

a The inscribed angle drawn in a semicircle is a/an .... angle.

1 obtuse 2 reflex 3 acute 4 right

b The number of common tangents for the two tangent circles externally is.................... .

1 1 2 3 3 4 4 2

c In the cyclic quadrilateral, each two opposite angles are ....

1 intersecting 2 supplementary 3 equal 4 corresponding

d If M and N are two touching externally circles with radii lengths 9 cm, and r cm respectively, if MN = 14 cm, then r = ... cm.

1 9 2 15 3 5 4 8

e The length of the opposite arc to the inscribed angle of measure 60° = ... circumference of the circle.

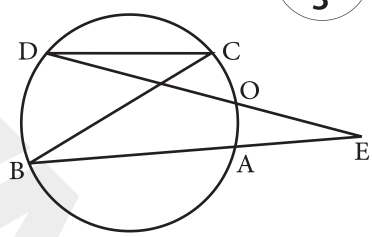
1 $\frac{1}{6}$ 2 $\frac{1}{3}$ 3 $\frac{1}{2}$ 4 otherwise

2 In the opposite figure:

3

E is a point outside the circle.

Prove that: m(∠DCB)>m(∠E).



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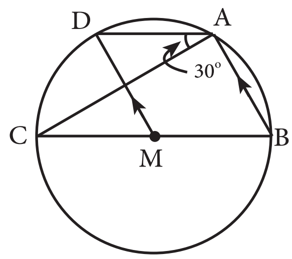
3 In the opposite figure:

3

$\overline{BC}$ is a diameter of circle M,

$\overline{AB}$ // $\overline{DM}$, m(∠DAC) = 30°

Find m(∠ACB).



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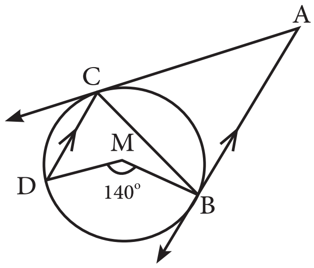
4 In the opposite figure:

3

$\overline{AB}$ and $\overline{AC}$ are two tangents to the circle M,

at B and C , $\overline{AB} \parallel \overline{CD}$, m $(\angle BMD) = 140^\circ$

Find: m$(\angle A)$

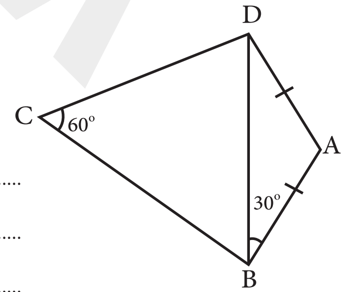


5 In the opposite figure:

3

a If AB = AD , m$(\angle ABD) = 30^\circ$, m$(\angle C) = 60^\circ$

b Prove that: ABCD is a cyclic quadrilateral.



15 Marks

Model (3)

1 Choose the correct answer:

3

a The figure which the circle doesn't pass through its vertices is a ....

1 square 2 triangle 3 rhombus 4 rectangle

b The measure of an arc which represents $\frac{1}{3}$ of the measure of the circle equals.......... .

1 180° 2 45° 3 120° 4 60°

c The length of the arc which represents $\frac{1}{4}$ of circumference of a circle = ...

1 πr 2 $\frac{1}{4}\pi r$ 3 $\frac{1}{2}\pi r$ 4 $2\pi r$

d The number of common tangents for the two intersecting circles is ....

1 0 2 3 3 infinite number 4 2

e In the circle M, the measure of the angle of tangency (ABL) ....
the measure of the ($\angle$AMB)

1 $\frac{1}{2}$ 2 2 3 1 4 4

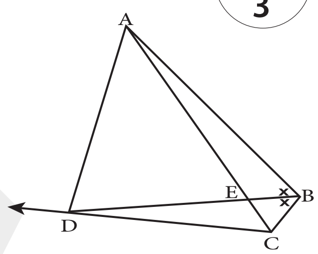
2 In the opposite figure:

3

ABCD is a cyclic quadrilateral, $\overrightarrow{BD}$ bisects $\angle$ABC,

If $\overline{BD} \cap \overline{AC} = \{E\}$,

prove that: $\overrightarrow{CD}$ is a tangent to the circle passing through the vertices of Δ BEC.



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3 In the opposite figure:

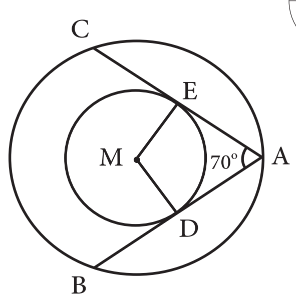
3

Two concentric circles at M,

$\overline{AB}$ and $\overline{AC}$ are two tangent segments to the smaller circle, m($\angle$A) = 70°

a Find: m($\angle$DME)

b Prove that: AB = AC



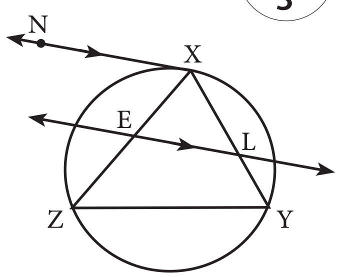
4 **In the opposite figure:**

3

XYZ is an inscribed triangle in a circle, if $L \in \overline{XY}$

And $\overleftrightarrow{LE}$ is drawn parallel to the tangent $\overrightarrow{XN}$ which touches the circle at X and intersects $\overline{XZ}$ at E.

Prove that: LYZE is a cyclic quadrilateral.



5 **In the opposite figure:**

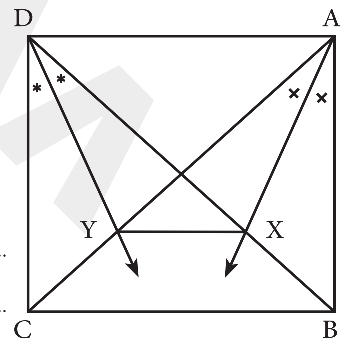
3

ABCD is a square, $\overrightarrow{AX}$ bisects $\angle BAC$,

and $\overrightarrow{DY}$ bisects $\angle CDB$.

a Prove that: the figure AXYD is a cyclic quadrilateral.

b Find with proof m ($\angle DXY$).



15 Marks

Model (1)

1 Choose the correct answer:

3

a The sum of measures of angles in the circle with radius r is ....

1 πr 2 270° 3 $2\,\pi r$ 4 **360**

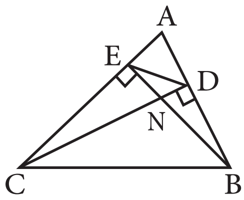
b The center of the inscribed circle of a triangle is the intersection point of its ....

1 medians 2 axes of symmetry of its sides

3 altitudes 4 **bisectors of its interior angles**

c In the opposite figure:

How many cyclic quadrilaterals are there? ....



1 4 2 **2**

3 1 4 3

d The diameter length of a circle is 8 cm. If the straight line L is at a distance of 4 cm from the center, then the straight line L is ....

1 a secant to the circle 2 an axis of symmetry to the circle

3 **a tangent to the circle** 4 outside the circle

e The measure of the exterior angle at any vertex of a cyclic quadrilateral .... the measure of the interior angle at the opposite vertex.

2 > 2 < 3 **=** 4 ≥

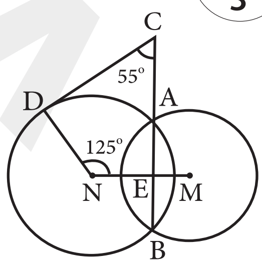
2 In the opposite figure:

3

M and N are two intersecting circles at A and B,

$C \in \overline{BA}$, $D \in$ the circle N, m ($\angle$MND) = 125° and m ($\angle$BCD) = 55°,

Prove that: $\overleftrightarrow{CD}$ is a tangent to the circle N at D.



$\because$ $\overleftrightarrow{MN}$ is the line of centers of two circles M and N,

$\overline{AB}$ is the common chord.

$\therefore \overline{AB} \perp \overline{MN}$

$\therefore$ m ($\angle$AEN) = 90°

$\because$ The sum of the measures of the interior angles of the quadrilateral CDNE = 360°

$\therefore$ m ($\angle$CDN) = $360^\circ - (55^\circ + 125^\circ + 90^\circ) = 90^\circ$

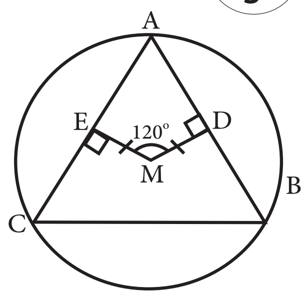
$\therefore \overline{ND} \perp \overline{CD}$

$\therefore$ $\overleftrightarrow{CD}$ is a tangent to the circle N at D.

3 **In the opposite figure:** 3

A circle M, $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$, where MD = ME,

m($\angle$DME) = 120°, prove that: the triangle ABC is equilateral.



$\because$ $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

, MD = ME $\therefore$ AB = AC (1)

From the quadrilateral ADME

m ($\angle$A) = 360° - (120° + 90° + 90°) = 60° (2)

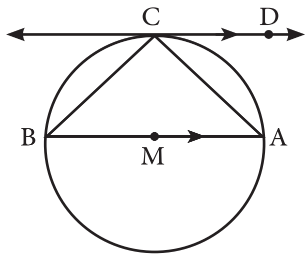
from (1) and (2):

$\therefore$ Δ ABC is an equilateral triangle.

4 **In the opposite figure:** 3

$\overline{CD}$ is a tangent to the circle M at C, $\overrightarrow{CB}$ // $\overline{BA}$

Prove that: m ($\angle$DCA) = 45°



$\because$ $\overleftrightarrow{CD}$ // $\overline{AB}$

$\therefore$ m ($\overset{\frown}{AC}$) = m($\overset{\frown}{BC}$)

, $\because$ $\overline{AB}$ is a diameter in circle M

$\therefore$ m ($\overset{\frown}{ACB}$) = 180°

$\therefore$ m ($\overset{\frown}{AC}$) = 180° ÷ 2 = 90°

, $\because$ m ($\angle$DCA) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$)

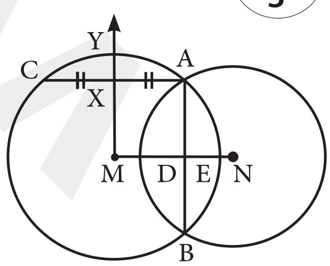
$\therefore$ m ($\angle$DCA) = $\frac{1}{2}$ x 90° = 45°

5 **In the opposite figure:** 3

If M and N are two intersecting circles at A and B,

AB = AC, X is the midpoint of $\overline{AC}$.

Prove that: XY = DE



$\because$MN is the line of centers of two circles M and N, $\overline{AB}$ is the common chord of the two circles.

$\therefore$ $\overline{MN} \perp \overline{AB}$

, $\because$ X is the midpoint of $\overline{AC}$

$\therefore$ $\overline{MX} \perp \overline{AC}$

$\because$ AB = AC $\therefore$ MX = MD (1) $\because$ MY = ME(lengths of two radii) (2)

by subtracting (1) from (2)

$\therefore$ MY - MX = ME - MD $\therefore$ XY = DE

15 Marks

Model (2)

1 Choose the correct answer: 3

a The inscribed angle drawn in a semicircle is a/an .... angle.

1 obtuse 2 reflex 3 acute 4 right

b The number of common tangents for the two tangent circles externally is.......................... .

1 1 2 3 3 4 4 2

c In the cyclic quadrilateral, each two opposite angles are ....

1 intersecting 2 supplementary 3 equal 4 corresponding

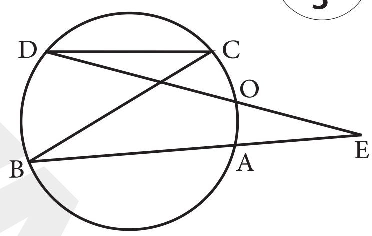
d If M and N are two touching externally circles with radii lengths 9 cm, and r cm respectively, if MN = 14 cm, then r = ... cm.

1 9 2 15 3 5 4 8

e The length of the opposite arc to the inscribed angle of measure 60˚ = ... circumference of the circle.

1 $\frac{1}{6}$ 2 $\frac{1}{3}$ 3 $\frac{1}{2}$ 4 otherwise

2 In the opposite figure: 3



E is a point outside the circle.

Prove that: m(∠DCB)>m(∠E).

$\because$ m (∠E) = $\frac{1}{2}$ {m ($\overset\frown{BD}$) - m ($\overset\frown{AO}$)}

$\therefore$ m (∠E) = $\frac{1}{2}$ m ($\overset\frown{BD}$) – $\frac{1}{2}$ m ($\overset\frown{AO}$)

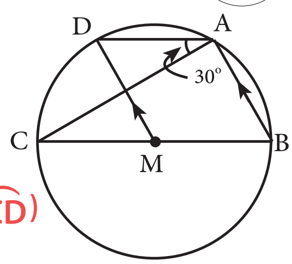
, $\because$ m (∠DCB) = $\frac{1}{2}$ m($\overset\frown{BD}$)

$\therefore$ m (∠E) = m (∠DCB) – $\frac{1}{2}$ m ($\overset\frown{AO}$)

$\therefore$ m (∠DCB) = m (∠E) + $\frac{1}{2}$ m ($\overset\frown{AO}$)

$\therefore$ m (∠DCB) > m (∠E)

3 In the opposite figure: 3



$\overline{BC}$ is a diameter of circle M,

$\overline{AB}$ // $\overline{DM}$, m(∠DAC) = 30˚

Find m(∠ACB).

$\because$ m (∠DMC) = 2m (∠CAD) (central and inscribed angles subtended by $\overset\frown{CD}$)

$\therefore$ m(∠DMC) = 2 × 30˚ = 60˚

$\because$ $\overline{AB}$ // $\overline{DM}$, $\overline{BC}$ is a transversal.

$\therefore$ m (∠B) = m(∠DMC) = 60˚(corresponding angles)

∵ $\overline{BC}$ is a diameter of circle M

∴ m (∠BAC) = 90°

∴ In Δ ABC: m (∠ACB) = 180° - (90° + 60°) = 30°

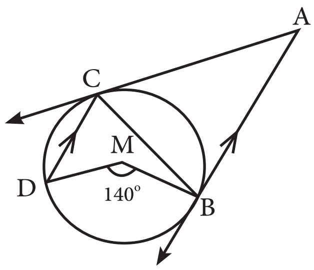
4 In the opposite figure:

3

$\overline{AB}$ and $\overline{AC}$ are two tangents to the circle M,

at B and C , $\overline{AB}$ // $\overline{CD}$, m (∠BMD) = 140°

Find: m(∠A)



∵ m (∠BCD) = $\frac{1}{2}$ m (∠BMD)(inscribed and central angles subtended by $\overset{\frown}{BD}$)

∴ m (∠BCD) = $\frac{1}{2}$ × 140° = 70°

, ∵ $\overline{AB}$ // $\overline{CD}$, BC is a transversal.

∴ m (∠ABC) = m (∠BCD) = 70° (alternate angles)

∵ $\overline{AB}$ and $\overline{AC}$ are two tangents to the circle from the same point.

∴ AB = AC

, ∵ AB = AC

∴ m (∠ABC) = m (∠ACB) = 70°

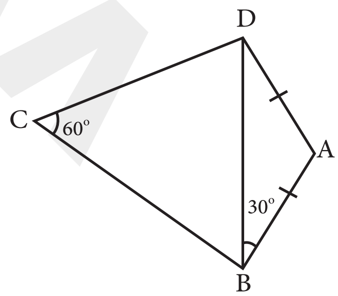
∴ In Δ ABC: m (∠A) = 180° - (2×70°) = 40°

5 In the opposite figure:

3

a If AB = AD , m(∠ABD) = 30° , m(∠C) = 60°

b Prove that: ABCD is a cyclic quadrilateral.



∴ In Δ ABC:

∵ AB = AD

∴ m(∠ABD) = m(∠ADB) = 30°

∴ m(∠A) = 180° - (2 × 30°) = 120°

, ∵ m(∠A) + m(∠C) = 120° + 60° = 180°

∴ ABCD is a cyclic quadrilateral.

15 Marks

Model (3)

1 Choose the correct answer:

3

a The figure which the circle doesn't pass through its vertices is a ....

1 square 2 triangle 3 rhombus 4 rectangle

b The measure of an arc which represents $\frac{1}{3}$ of the measure of the circle equals........... .

1 180° 2 45° 3 120° 4 60°

c The length of the arc which represents $\frac{1}{4}$ of circumference of a circle = ...

1 πr 2 $\frac{1}{4}\pi r$ 3 $\frac{1}{2}\pi r$ 4 $2\pi r$

d The number of common tangents for the two intersecting circles is ....

1 0 2 3 3 infinite number 4 2

e In the circle M, the measure of the angle of tangency (ABL) .... the measure of the ($\angle$AMB)

1 $\frac{1}{2}$ 2 2 3 1 4 4

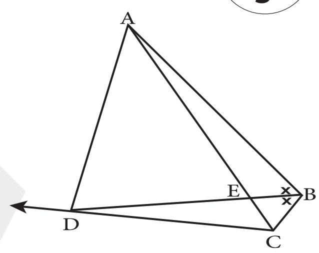
2 In the opposite figure:

3

ABCD is a cyclic quadrilateral, $\overrightarrow{BD}$ bisects $\angle$ABC,

If $\overline{BD} \cap \overline{AC} = \{E\}$,

prove that: $\overrightarrow{CD}$ is a tangent to the circle passing through the vertices of Δ BEC.



$\because$ ABCD is a cyclic quadrilateral.

$\because$ m($\angle$DCA) = m ($\angle$DBA) (1) (drawn on common base $\overline{AD}$ and on the same side of it)

, $\because$ $\overrightarrow{BD}$ bisects $\angle$ABC

$\therefore$ m($\angle$DBC) = m ($\angle$DBA) (2) (Given)

From (1) and (2): $\therefore$ m ($\angle$DBC) = m ($\angle$DCA)

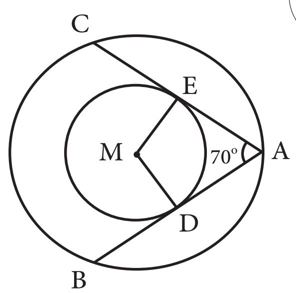
$\therefore$ $\overrightarrow{CD}$ is a tangent to the circle passing through the vertices of Δ BEC.

3 In the opposite figure:

3

Two concentric circles at M,

$\overline{AB}$ and $\overline{AC}$ are two tangent segments to the smaller circle, m($\angle$A) = 70°



a Find: m($\angle$DME)

b Prove that: AB = AC

∵ AB and AC are two tangents to the smaller circle

∴ $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

∴ m (∠MDA) = m (∠MEA) = 90°

∴ From the quadrilateral ADME:

m (∠DME) = 360° - (90° + 70° + 90°) = 110°

∵ MD = ME(two radii in the smaller circle), ∴ $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

∴ AB = AC

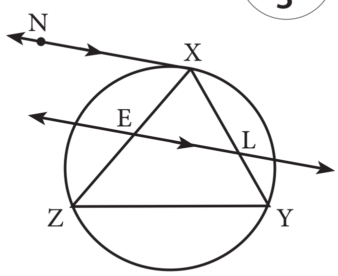
4 **In the opposite figure:**

3

XYZ is an inscribed triangle in a circle, if L ∈ $\overline{XY}$

And $\overleftrightarrow{LE}$ is drawn parallel to the tangent $\overrightarrow{XN}$ which touches the circle at X and intersects $\overline{XZ}$ at E.

Prove that: LYZE is a cyclic quadrilateral.



∵ $\overline{LE}$ // $\overrightarrow{XN}$, $\overleftrightarrow{XZ}$ is a transversal.

∴ m (∠XEL) = m (∠NXZ) (alternate angles)

∵ m (∠Y) (inscribed) = m (∠NXZ) (angle of tangency)

∴ m (∠Y) = m (∠XEL)

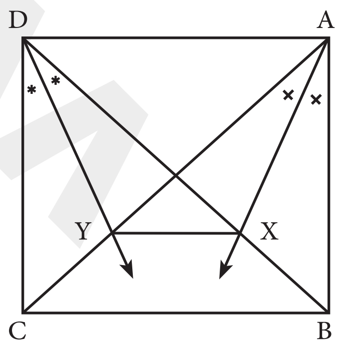
∴ The figure LYZE is a cyclic quadrilateral.

5 **In the opposite figure:**

3

ABCD is a square, $\overrightarrow{AX}$ bisects ∠BAC,

and $\overrightarrow{DY}$ bisects ∠CDB.



a Prove that: the figure AXYD is a cyclic quadrilateral.

b Find with proof m (∠DXY).

∵ ABCD is a square, $\overline{AC}$ and $\overline{BD}$ are two diagonals of the square.

∴ m (∠BAC) = m (∠BDC)

∴ $\frac{1}{2}$ m (∠BAC) = $\frac{1}{2}$ m (∠BDC)

∴ m (∠XAY) = m (∠XDY), but they are drawn on common base $\overline{XY}$ and on one side of it.

∴ The figure AXYD is a cyclic quadrilateral.

∴ m (∠DXY) = m (∠DAY) = 45° (they are drawn on common base $\overline{DY}$ and on one side of it)

Al-Adwaa Model 1

Algebra

1 Choose the correct answer:

(1) The simplest form of $n(x) = \frac{x+3}{x-3} \times \frac{x-3}{x^2-9}$ is ....

(a) $\frac{1}{x+3}$ (b) $\frac{1}{x-3}$ (c) $x+3$ (d) $x-3$

(2) If A and B are two mutually exclusive events, then P(A ∪ B) = ...

(a) P(B) (b) P(A ∩ B) (c) P(A) + P (B) (d) P(A)

(3) The set of zeroes of the function $f(x) = x(x^2 - 2x + 1)$ is ....

(a) {0 ,1} (b) {0, -1} (c) {0} (d) {1}

(4) The ordered pair which satisfies each of the following equations: x y = 2, x – y = 1 is ....

(a) (1, 2) (b) (2, 1) (c) (1 ,1) (d) (3, 1)

(5) The domain of the function $f: f(x) = \frac{2-x}{7}$ is ....

(a) $\mathbb{R} - \{7\}$ (b) $\mathbb{R} - \{2, 7\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R}$

(6) The domain of $n : n(x) = \frac{3x+4}{x^2+25} + \frac{x-2}{x^2+7}$ is ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5, 5, -7\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{-5, 5\}$

2

(a) Simplify: $n(x) = \frac{x}{x-2} \div \frac{x+3}{x^2-x-2}$,showing the domain of n

(b) Solve the following two equations in $\mathbb{R} \times \mathbb{R}$: $2x = 1 - y$, $x + 2y = 5$

3

(a) If A and B are two events of a random experiment and P(A) = 0.7 , P(A ∩ B) = 0.3, **find:** P(A – B)

(b) Simplify: $n(x) = \frac{x^2+x}{x^2-1} - \frac{x+5}{x^2+4x-5}$,showing the domain of n

4

(a) Find in $\mathbb{R}$ the solution set of the following equation by using the general formula:

$x^2 - 4x + 1 = 0$ approximating the result to two decimal places.

(b) If $n_1(x) = \frac{2x}{2x+6}$,$n_2(x) = \frac{x^2+3x}{x^2+6x+9}$, **prove that:** $n_1 = n_2$:

5

(a) If $n(x) = \frac{x-2}{x+1}$

Find: (1) the domain of n^{-1} (2) $n^{-1}(3)$

(3) If $n^{-1}(x) = 2$, **find** the value of x.

(b) Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations: $x - y = 1$,$x^2 - y^2 = 25$

Al-Adwaa Model 2

1 Choose the correct answer:

(1) The set of zeroes of the function $f : f(x) = x(x^2 - 1)$ is ....

(a) $\{0\}$ (b) $\{0, -1\}$ (c) $\{0, 1, -1\}$ (d) $\{0, 1\}$

(2) The set of zeroes of the function $f : f(x) = \frac{x^2 - 9}{x - 2}$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\{-3, 3\}$ (c) $\{2\}$ (d) $\{3, -3, 2\}$

(3) If $n(x) = \frac{x - 2}{x + 5}$, then the domain of n^{-1} is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{2, -5\}$ (d) $\mathbb{R} - \{-5\}$

(4) The common domain of the two fractions $\frac{2}{x^2 - 1}$ and $\frac{5x}{x^2 - x}$ is ....

(a) $\mathbb{R} - \{1\}$ (b) $\mathbb{R} - \{0, 1\}$ (c) $\mathbb{R} - \{0, 1, -1\}$ (d) $\mathbb{R} - \{1, -1\}$

(5) The S.S. of the two equations: $x - y = 0$, $x^2 + y^2 = 18$ in $\mathbb{R} \times \mathbb{R}$ is ....

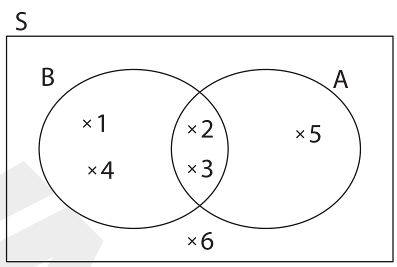
(a) $\{(3, 3)\}$ (b) $\{(-3, -3)\}$ (c) $\{(3, -3), (-3, 3)\}$ (d) $\{(3, 3), (-3, -3)\}$

(6) The set of zeroes of the function $f : f(x) = x^2 - 25$ is ....

(a) $\{5\}$ (b) $\{-5\}$ (c) $\{-5, 5\}$ (d) $\varnothing$

2

(a) If A and B are two events of a random experiment, then **find**:



(1) P (A ∩ B)

(2) P (A – B)

(3) The probability of non-occurrence of event A

(b) Simplify: $n(x) = \frac{x - 3}{x^2 - 7x + 12} - \frac{4}{x^2 - 4x}$, showing the domain of n.

3

(a) Find in $\mathbb{R}$ the solution set of the following equation by using the general rule:

$3x^2 - 5x - 4 = 0$ approximating the result to the nearest two decimal places.

(b) If the domain of the algebraic fraction $n : n(x) = \frac{x + 2}{x^2 + ax + b}$ is $\mathbb{R} - \{2, 3\}$.

Find the value of a and b.

4

(a) If $n(x) = \frac{x^2 - 3x}{(x - 3)(x^2 + 2)}$, then find $n^{-1}(x)$ in the simplest form, showing the domain of n^{-1}

(b) Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations: $x + y = 7$, $x^2 + y^2 = 25$

5

(a) Simplify: $n(x) = \frac{x^2 + 3x}{x^2 - 9} \div \frac{2x}{x + 3}$, showing the domain of n

(b) Solve the following two equations in $\mathbb{R} \times \mathbb{R}$: $x + 3y = 7$, $5x - y = 3$

Al-Adwaa Model 3

1 Choose the correct answer:

(1) If the S.S. of the two equations: $x + 2y = 5$ and $2x + k\,y = 3$ in $\mathbb{R} \times \mathbb{R}$ equals Ø, then k = ...

(a) 2 (b) –2 (c) 4 (d) –4

(2) Two positive numbers their sum is 9 and their product is 8, then the two numbers are ....

(a) 2,7 (b) 3,6 (c) 4 ,5 (d) 1,8

(3) If A is an event in the sample space of the random experiment, then P (A`) = ...

(a) 1 (b) –1 (c)1 – P (A) (d) P (A)–1

(4) If A and B are two events in a sample space for a random experiment $A \subset B$, then $P(A \cap B)$ = ...

(a) P (B) (b) P (A) (c) zero (d) $\varnothing$

(5) The solution set of the two equations: $x - y = 3$, $x + y = 7$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) {(6, 3)} (b) {(4 ,3)} (c) {(5, 2)} (d) {(3, 7)}

(6) If {3} is the solution set of the equation: $x^2 + mx = 3$, then m = ...

(a) –1 (b) –2 (c) 2 (d) 1

2

(a) Find in $\mathbb{R}$ the solution set of the following equation by using the general rule: $x^2 - 2x - 6 = 0$ approximating the result to one decimal places.

(b) A rectangle with a length more than its width by 4 cm if the perimeter of the rectangle is 28 cm, **find** the area of the rectangle.

3

(a) If the set of zeroes of the function $f : f(x) = ax^2 + x + b$ is $\{0, 1\}$ **find** the values of each two constants a and b.

(b) Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations: $y - x = 3$, $x^2 + y^2 - xy = 13$

4

(a) Simplify: $n(x) = \dfrac{x^3 - 8}{x^2 + x - 6} \times \dfrac{x + 3}{x^2 + 2x + 4}$,showing the domain of n

(b) Solve the following two equations in $\mathbb{R} \times \mathbb{R}$: $2x - y = 3$, $x + 2y = 4$

5

(a) If A and B are two events of a random experiment and P(A) = 0.3 , P(B) = 0.6 , $P(A \cap B) = 0.2$

Find: (1) $P(A \cup B)$ (2) P (A – B)

(b) Simplify: $n(x) = \dfrac{x^2 + 2x}{x^2 - 4} + \dfrac{x + 3}{x^2 - 5x + 6}$,showing the domain of n, then **find** $n(-2)$ if it is possible.

Al-Adwaa Model 1

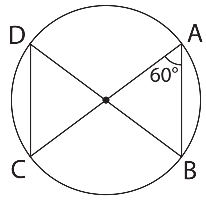
Geometry

1 Choose the correct answer:

(1) .... is a line segment with one endpoint at the centre of the circle and the other endpoint on the circle.

(a) Diameter (b) Radius (c) Chord (d) Axis of symmetry

(2) In the opposite figure:



If m ($\angle$ BAC) = 30°, then m ($\angle$ BDC) = ...

(a) 15° (b) 60°

(c) 30° (d) 90°

(3) The length of the arc opposite to a central angle of measure 30° in a circle of circumference 36 cm = ... cm

(a) 18 (b) 9 (c) 4.5 (d) 3

(4) M and N are two circles of radii lengths 9 cm, and 4 cm respectively MN = 5 cm, then the two circles are ....

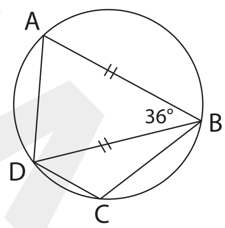
(a) touching externally (b) intersecting

(c) touching internally (d) distant

(5) The quadrilateral is cyclic if there is an exterior angle at any of its vertices .... the measure of the interior angle at the opposite vertex.

(a) greater than (b) complements (c) supplements (d) equal to

(6) In the opposite figure:

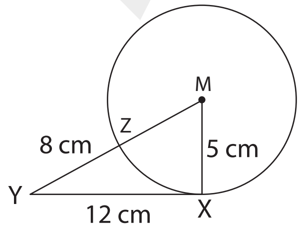


If AB = BD and m ($\angle$ ABD) = 36°, then m ($\angle$ C) = ...

(a) 140° (b) 54°

(c) 70° (d)108°

2 (a) In the opposite figure:



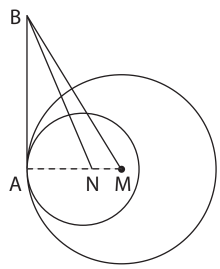
M is a circle with radius length 5 cm

XY = 12 cm, $\overline{MY} \cap$ circle M = {Z} and ZY = 8 cm.

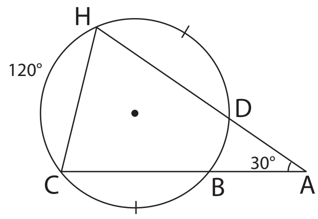
Prove that: $\overleftrightarrow{XY}$ is a tangent to circle M at X.

(b) M and N are two circles with radii lengths of 10 cm and 6 cm respectively and they are touching internally at A, $\overleftrightarrow{AB}$ is a common tangent for both at A.

If the area of the triangle BMN = 24 cm^2. **Find** the length of $\overline{AB}$.



3 **(a) In the opposite figure:**

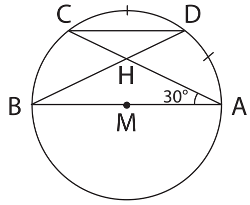


m ($\angle$ A) = 30°, m ($\overset{\frown}{HC}$) = 120°, m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DH}$)

(1) **Find:** m ($\overset{\frown}{BD}$) the minor

(2) **Prove that:** AB = AD

(b) In the opposite figure:



$\overline{AB}$ is a diameter in the circle M

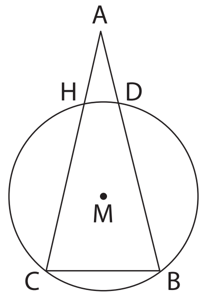
, C $\in$ the circle M, m ($\angle$ CAB) = 30°

, D is midpoint of $\overset{\frown}{AC}$, $\overline{DB} \cap \overline{AC} = \{H\}$

(1) **Find:** m ($\angle$ BDC) and m ($\overset{\frown}{AD}$)

(2) **Prove that:** $\overline{AB}$ // $\overline{DC}$

4 **(a) In the opposite figure:**

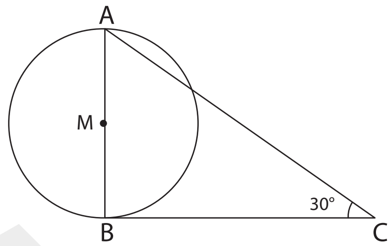


ABC is a triangle in which AB = AC , $\overline{BC}$ is a chord

in the circle M , if $\overline{AB}$ and $\overline{AC}$ cut the circle at D and H respectively.

Prove that: m ($\overset{\frown}{DB}$) = m ($\overset{\frown}{HC}$)

(b) In the opposite figure:

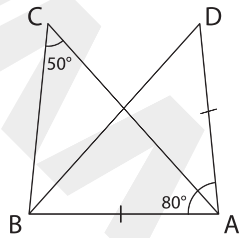


A circle M of circumference 44 cm, $\overline{AB}$ is a diameter,

$\overline{BC}$ is a tangent at B and m ($\angle$ ACB) = 30°

Find the length of $\overline{BC}$ $\left(\pi = \frac{22}{7}\right)$

5 **(a) In the opposite figure:**



AB = AD , m ($\angle$ DAB) = 80° , m ($\angle$ C) = 50°

Prove that: The points A, B, C and D

have a circle passing through them.

(b) Mention two cases of the cyclic quadrilateral.

Al-Adwaa Model 2

1 Choose the correct answer:

(1) If M is a circle, its diameter length is 6 cm, and A is a point on the circle, then ……………

(a) MA > 6 cm (b) MA = 6 cm (c) MA = 3 cm (d) MA < 3 cm

(2) The type of the inscribed angle which is opposite to an arc greater than a semicircle is …………… angle.

(a) an acute (b) an obtuse (c) a right. (d) a straight

(3) $\overline{AB}$ and $\overline{CD}$ are two chords in a circle, AB = 5 cm and CD = 3 cm, then the chord which is nearer to the centre of the circle is ……………

(a) $\overline{AB}$ (b) $\overline{CD}$

(c) both are equal (d) cannot be determined

(4) We can identify the circle if we are given ……………

(a) three collinear points (b) two points

(c) three non-collinear points (d) one point

(5) If the measure of an angle of tangency = 70°, then the measure of the central angle subtended by the same arc equals ……………

(a) 35° (b) 70° (c) 140° (d) 105°

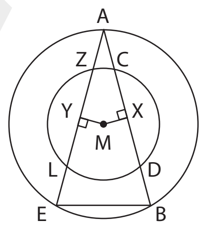
(6) The measure of the inscribed angle which is drawn in $\frac{1}{6}$ of a circle equals ……………

(a) 240° (b) 120° (c) 60° (d) 30°

2

(a) Two concentric circles M, $\overline{AB}$ is a chord in the larger circle and intersects the smaller circle at C and D, $\overline{AE}$ is a chord in the larger circle and intersects the smaller circle at Z and L, $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AE}$

If m (∠ ABE) = m (∠ AEB), then **prove that:** CD = ZL.

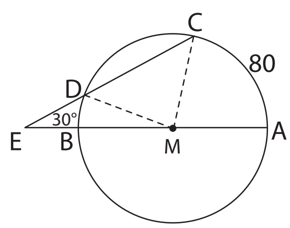


(b) In the opposite figure:

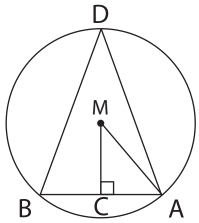
$\overline{AB}$ is a diameter in the circle M, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$,

m (∠ AEC) = 30°, m $(\overset{\frown}{AC})$ = 80°

Find: m $(\overset{\frown}{CD})$



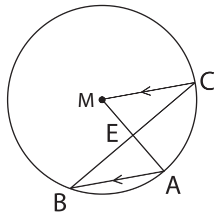
3 **(a) In the opposite figure:**



$\overline{AB}$ is a chord of circle M, $\overline{MC} \perp \overline{AB}$.

Prove that: m ($\angle$ AMC) = m ($\angle$ ADB)

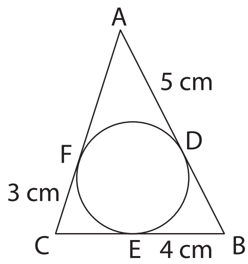
(b) In the opposite figure:



$\overline{AB}$ is a chord in circle M, $\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$,

Prove that: BE > AE.

4 **(a) In the opposite figure:**

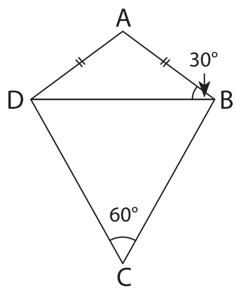


A circle is drawn touching the sides of a triangle ABC

, $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D, E, F, AD = 5 cm, BE = 4 cm, CF = 3 cm

Find the perimeter of Δ ABC

(b) In the opposite figure:

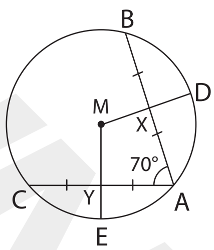


ABCD is a quadrilateral in which AB = AD,

m ($\angle$ ABD) = 30°, m ($\angle$ C) = 60°

Prove that: ABCD is a cyclic quadrilateral.

5 **(a) In the opposite figure:**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length at the circle M

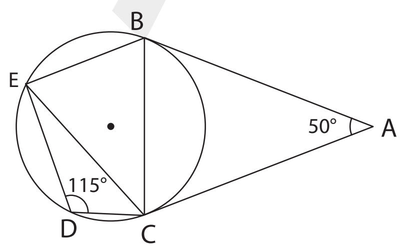
, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m ($\angle$ A) = 70°

(1) **Find:** m ($\angle$ DME)

(2) **Prove that:** XD = YE

(b) In the opposite figure:



$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle at B and C

, m ($\angle$ A) = 50° , m ($\angle$ D) = 115°

Prove that:

(1) $\overrightarrow{BC}$ bisects $\angle$ ABE (2) CB = CE

Al-Adwaa Model 3

1 Choose the correct answer:

(1) If the chords of a circle are equal in length, then they are .... the centre.

(a) passing through (b) equidistant from

(c) intersecting at (d) perpendicular to

(2) The central angle whose measure is 90° subtended by an arc of length = ... the circumference of the circle.

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(3) The length of the arc that is opposite to a right inscribed angle in a circle whose circumference is 44 cm equals .... cm.

(a) 22 (b) 11 (c) $\frac{22}{7}$ (d) $\frac{44}{7}$

(4) Any straight line passing through the centre of the circle is called .... of it.

(a) diameter (b) radius (c) chord (d) axis of symmetry

(5) The number of common tangents of two non-congruent concentric circles is ....

(a) 1 (b) 2 (c) 4 (d) zero

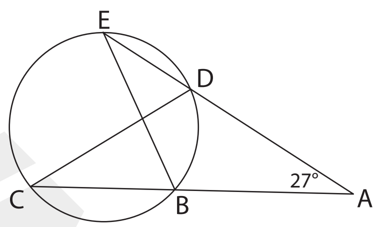
(6) In the opposite figure:

$\overrightarrow{AD}$ intersects the circle at D and E, $\overrightarrow{AB}$ intersects it at B and C.

If m (∠ A) = 27°, AB = BE, then m (∠ CDE) = ...

(a) 13.5° (b) 54°

(c) 27° (d) 36°

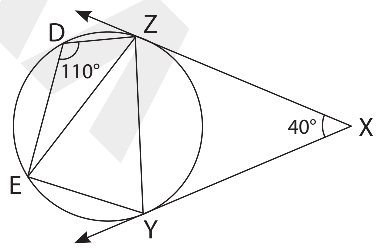


2 (a) In the opposite figure:

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle at the two point Y and Z, m (∠ X) = 40°

, m (∠ D) = 110°

Prove that: m (∠ ZYE) = m (∠ ZEY)

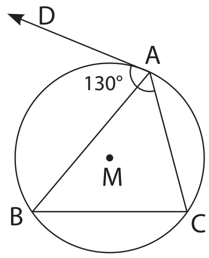


(b) In the opposite figure:

$\overrightarrow{AD}$ is the tangent to the circle M at A

, m (∠ DAC) = 130°

Find with proof: m (∠ B)

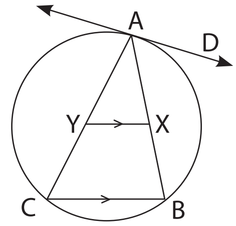


3 **(a)** ABC is a triangle inscribed in a circle,

$\overleftrightarrow{AD}$ is a tangent to the circle at A, $X \in \overline{AB}$, $Y \in AC$ where $\overline{XY} \parallel \overline{BC}$

Prove that:

$\overline{AD}$ is a tangent to the circle passing through the points A, X and Y .



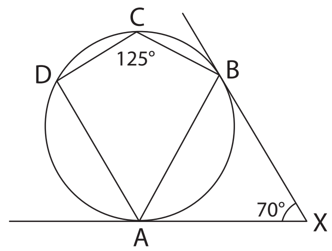
(b) $\overline{XA}$ and $\overline{XB}$ are two tangents to the circle at A and B

m ($\angle$ AXB) = 70°, m ($\angle$ DCB) = 125°

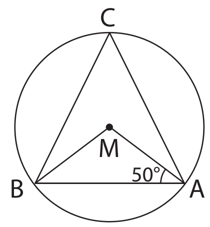
Prove that:

First: $\overrightarrow{AB}$ bisects $\angle$ DAX .

Second: $\overline{AD} \parallel \overline{XB}$.



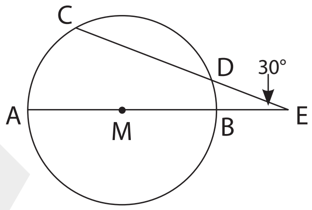
4 **(a)** M is a circle , m ($\angle$ MAB) = 50° , find m ($\angle$ C).



(b) In the opposite figure:

$\overline{AB}$ is a diameter in the circle M, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$,

m ($\angle$ E) = 30° , m ($\overset{\frown}{AC}$) = 80° , find m ($\overset{\frown}{BD}$)



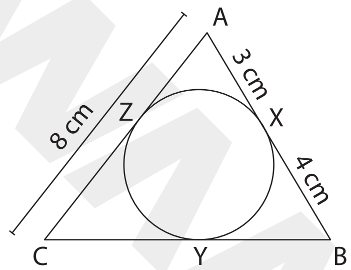
5 **(a) In the opposite figure:**

An inscribed circle of triangle ABC

touches its sides at X , Y and Z.

If AX = 3cm , XB = 4 cm , AC = 8 cm,

find the length of $\overline{BC}$.



(b) In the figure opposite:

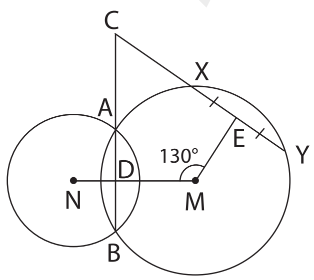
M and N are two intersecting circles

where circle M ∩ circle N equal {A , B}

$\overrightarrow{YC} \cap \overrightarrow{BA} = \{C\}$

If E is the midpoint of $\overline{XY}$,

m ($\angle$ EMN) = 130°, **find** m ($\angle$ C).



Model Answer 1

Algebra

1 Choose the correct answer:

(1) $\frac{1}{x-3}$

(2) P (A) + P (B)

(3) $\{0 , 1\}$

(4) (2 , 1)

(5) $\mathbb{R}$

(6) $\mathbb{R}$

2 (a) $n(x) = \frac{x}{x-2} \div \frac{x+3}{(x-2)(x+1)}$ factorize

Domain of $n = \mathbb{R} - \{2 , -1 , -3\}$

$n(x) = \frac{x}{x-2} \times \frac{(x-2)(x+1)}{x+3}$ switch to multiplication

$n(x) = \frac{x(x+1)}{x+3}$ simplify

(b) $y = (1 - 2x)$ (1)

$x + 2y = 5$ (2)

By substituting (1) in (2)

$x + 2(1 - 2x) = 5$

$x + 2 - 4x = 5$

$-3x = 3$

$x = -1$

Substitute in (1)

$y = (1 - 2(-1))$

$y = 3$

S.S. $= \{(-1 , 3)\}$

3 (a) $P(A - B) = P(A) - P(A \cap B)$

$P(A - B) = 0.7 - 0.3 = 0.4$

(b) $n(x) = \frac{x(x+1)}{(x-1)(x+1)} - \frac{x+5}{(x+5)(x-1)}$ factorize

Domain of n $= \mathbb{R} - \{1 , -1 , -5\}$

$n(x) = \frac{x}{(x-1)} - \frac{1}{(x-1)}$ simplify

$n(x) = \frac{x-1}{(x-1)}$ subtract

$n(x) = 1$ simplify

4 **(a)** a = 1 , b = –4 , c = 1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

$x = 2 + \sqrt{3} \simeq 3.73$, $x = 2 - \sqrt{3} \simeq 0.27$

S.S.= { 3.73 , 0.27 }

(b): $\because n_1(x) = \frac{2x}{2x+6} = \frac{2x}{2(x+3)} = \frac{x}{x+3}$

$\because$ The domain $n_1 = \mathbb{R} - \{-3\}$

$\because n_2(x) = \frac{x^2+3x}{x^2+6x+9} = \frac{x(x+3)}{(x+3)(x+3)} = \frac{x}{x+3}$

$\because$ The domain $n_2 = \mathbb{R} - \{-3\}$

$\because n_1(x) = n_2(x)$, domain of n_1 = domain of n_2

$\therefore n_1 = n_2$

5 **(a)** (1) $n(x) = \frac{x-2}{x+1}$

$n^{-1}(x) = \frac{x+1}{x-2}$

the domain of $n^{-1} = \mathbb{R} - \{-1, 2\}$

(2) $n^{-1}(3) = \frac{3+1}{3-2} = 4$

(3) $\because n^{-1}(x) = \frac{x+1}{x-2}$

$\therefore \frac{x+1}{x-2} = 2$

$2(x-2) = x + 1$

$2x - 4 = x + 1$

$2x - x = 1 + 4$

$\therefore x = 5$

(b) $x - y = 1$ (1)

$x^2 - y^2 = 25$

$(x - y)(x + y) = 25$ (2)

Substitute (1) in (2)

$(x + y) = 25$ (3)

By adding (1) and (3)

Substitute (1) in (2)

$2x = 26$

$x = 13$

Substitute in (1)

$13 - y = 1$

$y = 12$ S.S = {(13, 12)}

Model Answer 2

1 Choose the correct answer:

(1) $\{0,1,-1\}$ (2) $\{-3, 3\}$ (3) $\mathbb{R}-\{2,-5\}$

(4) $\mathbb{R}-\{0,1,-1\}$ (5) $\{(3, 3), (-3, -3)\}$ (6) $\{-5, 5\}$

2

(a) (1) $P(A \cap B)=\frac{2}{6}=\frac{1}{3}$

(2) $P(A-B)=\frac{1}{6}$

(3) $P(A`)=\frac{3}{6}=\frac{1}{2}$

(b) $n(x)=\frac{x-3}{(x-3)(x-4)}-\frac{4}{x(x-4)}$ factorize

Domain $n=\mathbb{R}-\{3,4,0\}$

$n(x)=\frac{1}{(x-4)}-\frac{4}{x(x-4)}$ simplify

$n(x)=\frac{x}{x(x-4)}-\frac{4}{x(x-4)}$ common denominator

$n(x)=\frac{x-4}{x(x-4)}$ subtract

$n(x)=\frac{1}{x}$ simplify

3

(a) a = 3, b = –5 , c = –4

$$x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-5) \pm \sqrt{(-5)^2-4(3)(-4)}}{2(3)}$$

$$x=\frac{5 \pm \sqrt{73}}{6}$$

$$x=\frac{5+\sqrt{73}}{6} \simeq 2.26 \ , \ x=\frac{5-\sqrt{73}}{6} \simeq -0.59$$

S.S. $=\{2.26, -0.59\}$

(b) $\because$ The domain of the $n(x)$ is $\mathbb{R}-\{2,3\}$

$\therefore$ $n(2)$ and $n(3)$ are undefined

$\because n(2)=\frac{4}{2^2+2a+b}$

$4-2a+b=0 \quad \therefore 2a+b=-4$ (1)

$\because n(3)=\frac{5}{3^2+3a+b}$ Type equation here.

$9+3a+b=0 \quad \therefore 3a+b=-9$ (2)

$2a+b=-4$ $(x-1)$

$3a+b=-9$

$-2a-b=4$ (1)

$3a+b=-9$ (2) by adding (1) and (2)

$a=-5$ by substituting in the first equation $-10+b=-4$

$b=6$

4 **(a)** $n^{-1}(x) = \frac{(x-3)(x^2+2)}{x^2-3x} = \frac{(x-3)(x^2+2)}{x(x-3)}$

domain $= \mathbb{R} - \{0, 3\}$ $\quad n^{-1}(x) = \frac{(x^2+2)}{x}$

(b) $x = (7 - y)$ (1)

$x^2 + y^2 = 25$ (2)

Substitute (1) in (2)

$(7 - y)^2 + y^2 = 25$

$49 - 14y + y^2 + y^2 = 25$

$2y^2 - 14y + 49 = 25$

$2y^2 - 14y + 24 = 0$ $(\div 2)$

$y^2 - 7y + 12 = 0$

$(y - 4)(y - 3) = 0$

$y = 4$, $y = 3$ substitute in (1)

$x = 3$, $x = 4$

S.S. $= \{(3, 4), (4, 3)\}$

5 **(a)** $n(x) = \frac{x(x+3)}{(x+3)(x-3)} \div \frac{2x}{x+3}$ factorize

Domain of $n = \mathbb{R} - \{-3, 3, 0\}$

$n(x) = \frac{x(x+3)}{(x+3)(x-3)} \times \frac{x+3}{2x}$ switch to multiplication

$n(x) = \frac{x+3}{2(x-3)} = \frac{x+3}{2x-6}$ simplify

(b) $5x - y = 3$ $(\times 3)$

$15x - 3y = 9$ (1)

$x + 3y = 7$ (2)

By adding (1) , (2)

$16x = 16$

$x = 1$

Substitute in (2)

$1 + 3y = 7$

$3y = 6$

$y = 2$

S.S. $= \{(1,2)\}$

Model Answer 3

1 Choose the correct answer:

(1) 4

(2) 1,8

(3) 1 – P (A)

(4) P (A)

(5) $\{(5, 2)\}$

(6) –2

2 **(a)** $a = 1 , b = -2 , c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{28}}{2}$$

$x = 1 + \sqrt{7} \simeq 3.6 \quad , \quad x = 1 - \sqrt{7} \simeq -1.6$

S.S. = $\{3.6 , -1.6\}$

(b) Let the length = L and the width = W

$L = (W + 4) \qquad (1)$

$2 (L + W) = 28 \qquad (2)$

By substituting (1) in (2)

$2 (W + 4 + W) = 28$

$2W + 4 = 14$

$2W = 10$

$W = 5$ cm

By substituting in (1)

$L = 5 + 4 = 9$ cm

Area = $L \times W = 9 \times 5 = 45$ cm^2

3 **(a)** $\because$ Z (f) = {0 , 1} by substituting $x = 0$

$\therefore$ b = 0

substituting $x = 0$

$a + 1 + 0 = 0 \qquad a = -1$

(b) $y = (x + 3)$ (1)

$x^2 + y^2 - xy = 13$ (2)

Substitute (1) in (2)

$x^2 + (x + 3)^2 - x(x + 3) = 13$

$x^2 + x^2 + 6x + 9 - x^2 - 3x - 13 = 0$

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

$x = -4 , x = 1$ substitute in (1)

$y = -1 , \ y = 4$

S.S. $= \{(-4 , -1) , (1 , 4)\}$

4 **(a)** $n(x) = \frac{(x-2)(x^2+2x+4)}{(x+3)(x-2)} \times \frac{x+3}{(x^2+2x+4)}$ factorize

Domain of $n = \mathbb{R} - \{-3 , 2\}$

$n(x) = 1$ simplify

(b): $2x - y = 3$ $(\times 2)$

$4x - 2y = 6$ (1)

$x + 2y = 4$ (2)

By adding (1) , (2)

$5x = 10$

$x = 2$

Substitute in (2)

$2 + 2y = 4$

$2y = 2$

$y = 1$

S.S. $= \{(2 , 1)\}$

5 **(a)** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.3 + 0.6 - 0.2 = 0.7$

$P(A - B) = P(A) - P(A \cap B)$

$P(A - B) = 0.3 - 0.2 = 0.1$

(b) $n(x) = \frac{x(x+2)}{(x+2)(x-2)} + \frac{x+3}{(x-3)(x-2)}$ factorize

Domain of $n = \mathbb{R} - \{3 , 2 , -2\}$

$n(x) = \frac{x}{(x-2)} + \frac{x+3}{(x-3)(x-2)}$ simplify

$n(x) = \frac{x(x-3)}{(x-2)(x-3)} + \frac{x+3}{(x-3)(x-2)}$ common denominator

$n(x) = \frac{x^2 - 3x + (x+3)}{(x-2)(x-3)}$ add

$n(x) = \frac{x^2 - 2x + 3}{(x-2)(x-3)} = \frac{x^2 - 2x + 3}{x^2 - 5x + 6}$

$n(-2)$ is undefined because $-2 \notin$ the domain

Model Answer 1

Geometry

1 **Choose the correct answer:**

(1) radius. (2) 60° (3) 3

(4) Touching internally (5) equal to (6) 108°

2 **(a)** $\because \overline{MY} \cap$ circle $M = \{Z\}$

$\therefore MY = MZ + ZY$

$\because MZ = MX = 5$ cm (radii)

$\therefore MY = 5 + 8 = 13$ cm

$\because (MY)^2 = (13)^2 = 169$

$(MX)^2 = (5)^2 = 25$ $(XY)^2 = (12)^2 = 144$

$(MX)^2 + (XY)^2 = 25 + 144 + 169 = (MY)^2$

$\therefore m\angle MXY = 90°$ (The converse of the pythagoras' theorem)

$\because \overline{XY} \perp \overline{MX}$ and $\overline{MX}$ is a radius

$\therefore \overline{XY}$ is a tangent to the circle at X.

(b) $\because$ The two circles are touching internally at A

$\therefore A \in \overline{MN}$, $\overline{MN} \perp \overline{AB}$

$\because MN = 10 - 6 = 4$ cm (Touching internally)

$\because$ Area $\Delta BMN = \frac{1}{2} \times MN \times AB$

$\therefore 24 = 4 \times \frac{1}{2} \times AB$ $\therefore AB = 12$ cm

3 **(a)** $m(A) = \frac{1}{2}[m(\overset{\frown}{CH}) - m(\overset{\frown}{BD})]$

$30° = \frac{1}{2}[120 - m(\overset{\frown}{BD})]$

$60° = 120° - m(\overset{\frown}{BD})$

$m(\overset{\frown}{BD}) = 60°$

$m(\overset{\frown}{CH}) + m(\overset{\frown}{HD}) + m(\overset{\frown}{BD}) + m(\overset{\frown}{BC}) = 360°$

$\because m(\overset{\frown}{HD}) = m(\overset{\frown}{BC}) = \frac{360° - (120° + 60°)}{2}$

$m(\overset{\frown}{HD}) = m(\overset{\frown}{BC}) = 90°$

$\because \angle C$ is an inscribed angle subtended by $\overset{\frown}{HDB}$

$\because m(\angle C) = \frac{1}{2} m(\overset{\frown}{HDB}) = \frac{1}{2} \times 150° = 75°$

In ΔACH:

$m(\angle H) = 180° - (30° + 75°) = 75°$

$m(\angle H) = m(\angle HCB) = 75°$

and $AH = AC$ (1)

$m(\overset{\frown}{BC}) = m(\overset{\frown}{HD})$

$HD = BC$ (2)

By subtracting (2) from (1)

$AH - AB = AC - BC$ $AD = AB$

(b) $\because$ $\overline{AB}$ is a diameter

$\therefore$ m $(\overset{\frown}{AD})$ + m $(\overset{\frown}{CD})$ + m $(\overset{\frown}{BC})$ = 180°

m ($\angle$ A) = 30°

$\because$ m $(\overset{\frown}{BC})$ = 60°

$\because$ m $(\overset{\frown}{CD})$ + m $(\overset{\frown}{AD})$ = 180° – 60 ° = 120°

$\therefore$ m $(\overset{\frown}{AD})$ = m $(\overset{\frown}{CD})$ = $\frac{120°}{2}$ = 60°

m $\angle$ ABD = $\frac{1}{2}$ m $(\overset{\frown}{AD})$= 30°

m $\angle$ CDB = $\frac{1}{2}$ m $(\overset{\frown}{BC})$= 30°

$\because$ m ($\angle$ DBA) = m ($\angle$ CDB) = 30° They are alternate

$\therefore$ $\overline{AB}$ // $\overline{CD}$

4 **(a)** $\because$ AB = AC

$\therefore$ m ($\angle$B) = m ($\angle$C)

$\therefore$ m $(\overset{\frown}{DHC})$ = m $(\overset{\frown}{HDB})$

by subtracting m $(\overset{\frown}{HD})$ from both sides

$\therefore$ m $(\overset{\frown}{DB})$ = m $(\overset{\frown}{HC})$

(b) $\because$ circumference = 44 cm

$\therefore$ 2π r = 44

r = 44 ÷ (2 × $\frac{22}{7}$) = 7 cm

$\because$ $\overline{AB}$ is a diameter of length 14 cm and $\overline{BC}$ is a tangent

$\therefore$ m($\angle$B) = 90°

$\therefore$ m($\angle$ACB) = 30°

$\therefore$ AC = 28 cm and $(BC)^2 = (AC)^2 - (AB)^2$

BC = $\sqrt{28^2 - 14^2} = 14\sqrt{3}$ cm

5 **(a)** In Δ ABD

$\because$ AB = AD, m ($\angle$DAB) = 80°

$\therefore$ m($\angle$D) = m($\angle$ABD) = $\frac{180° - 80°}{2}$ = 50°

$\because$ m($\angle$D) = m ($\angle$C) = 50°

$\because$ They are both drawn angles on the same base $\overline{AB}$ and on one side of it.

$\because$ They points A, B , C and D have a circle passing through them.

(b) Mention any two cases of the following:

1-If there is a point in the plane equidistant from all vertices.

2- If there is an exterior angle its measure = the measure of the niterior angle at the opposite vertex.

3 - If there are two opposite angles are supplementary.

4- If there are two angles equal in measure and drawn on the same base and one side of this base.

Model Answer 2

1 Choose the correct answer:

(1) MA = 3 cm. (2) an obtuse (3) $\overline{AB}$

(4) three non-collinear points (5) 140° (6) 30°

2

(a) In Δ ABE:

$\because$ m ($\angle$ ABE) = m ($\angle$ AEB)

$\therefore$ AB = AE

In the larger circle:

$\because$ AB = AE

$\because$ $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AE}$

$\therefore$ MX = MY

In the smaller circle:

$\because$ MX = MY

$\because$ $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AE}$

$\therefore$ CD = ZL

(b) $\because$ $\overline{AB}$ is a diameter in circle M

$\therefore$ m ($\overset{\frown}{ACB}$) =180°

Draw ($\overline{MC}$) , ($\overline{MD}$)

$\because$ m ($\overset{\frown}{AC}$) = 80°

$\therefore$ m ($\angle$ AMC) = 80°

$\therefore$ m ($\angle$ CME) = 180° – 80° = 100°

In triangle Δ CME:

$\therefore$ m ($\angle$ ECM) = 180° – (30° + 100°) = 50°

In triangle Δ CMD:

$\because$ MC = MD radii

$\therefore$ m ($\angle$ CMD) = 180° – (50° + 50°) = 80°

$\therefore$ m ($\overset{\frown}{CD}$) = 80°

3

(a) Draw $\overline{BM}$

Proof:

In Δ MAB:

$\because$ $\overline{MA} = \overline{MB}$ (radii), $\overline{MC} \perp \overline{AB}$

$\therefore$ m ($\angle$ AMC) = m ($\angle$ BMC) = $\frac{1}{2}$ m ($\angle$ AMB) (1) isosceles triangle properties

$\because$ inscribed $\angle$ ADB and central $\angle$ AMB are subtended at ($\overset{\frown}{AB}$)

$\therefore$ m ($\angle$ ADB) = $\frac{1}{2}$ m ($\angle$ AMB) (2)

$\therefore$ From (1) and (2) we get: m ($\angle$ AMC) = m ($\angle$ ADB).

(b) $\because \overline{CM} // \overline{AB}$

$\therefore$ m ($\angle$ CMA) = m ($\angle$ MAB) alternate angles

$\because$ m ($\angle$ CMA) = 2 × m ($\angle$ CBA) central and inscribed

$\therefore$ m ($\angle$ MAB) = 2 × m ($\angle$ CBA)

$\therefore$ m ($\angle$ MAB) > m ($\angle$ CBA)

In Δ ABE: $\therefore$ BE > AE

4 **(a)** $\because \overline{AB}$, $\overline{BC}$ and $\overline{AD}$ are three tangents to the circle

$\therefore$ AD = AF = 5 cm.

BD = BE = 4 cm

CE = CF = 3 cm

$\because$ Perimeter of ⊿ABC = AB + BC + AC

$\therefore$ Perimeter of ⊿ABC = 8 + 9 + 7 = 24 cm.

(b) In ⊿ABC:

$\because$ AB = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ MDB) = 30°

m ($\angle$ BAD) = 180° – (30° + 30°) = 120°

m ($\angle$ A) + m ($\angle$ C) = 180° (opposite angles)

ABCD is a cyclic quadrilateral.

5 **(a) In the circle M:**

$\because$ X and y are midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

$\therefore \overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AC}$

And m ($\angle$ MXA) = m ($\angle$ MYA) = 90°

In the quadrilateral AXMY:

$\because$ m ($\angle$ A) + m ($\angle$ XMY) + m ($\angle$ MXA) + m ($\angle$ MYA) = 360°

$\therefore$ m ($\angle$ DME) = 360° – (90 ° + 90 ° + 70°) = 110°

$\because$ AB = AC

$\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{AC}$

$\therefore$ MX = MY $\because$ MD = ME = r

By subtracting MX from MD and MY from ME we get that: XD = YE

(b) $\overline{AB}$ and $\overline{AC}$ are two tangents

$\therefore$ AB = AC and m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 50°}{2}$ = 65°

$\because$ EBCD is a cyclic quadrilateral.

$\therefore$ m ($\angle$ EDC) + m ($\angle$ CBE) = 180°

m ($\angle$ CBE) = 180° – 115° = 65°

$\because$ m ($\angle$ ABC) = m ($\angle$ EBC) = 65°

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABE

$\because \angle$ BEC is an inscribed angle subtended by ($\overset{\frown}{BC}$) and $\angle$ ABC is an angle of tangency subtended by ($\overset{\frown}{BC}$)

$\therefore$ m ($\angle$ ABC) = m ($\angle$ BEC) = 65°

$\because$ m ($\angle$ EBC) = m ($\angle$ BEC) = 65°

$\therefore$ CB = CE

Model Answer 3

1 Choose the correct answer:

(1) equidistant from (2) $\frac{1}{4}$ (3) 22

(4) Axis of symmetry (5) zero (6) 54°

2

(a) $\because \overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents

$\therefore XY = XZ$

And $m(\angle xzy) = m(\angle xyz) = \frac{180° - 40°}{2} = 70°$

$\therefore$ ZYED is a cyclic quadrilateral.

$\therefore m(\angle ZDE) + m(\angle ZYE) = 180°$

And $m(\angle EYZ) = 180° - 110° = 70°$ (1)

$\because \angle ZEY$ is an inscribed angle subtended by $(\overset{\frown}{ZY})$

and $\angle XZY$ is an angle of tangency subtended by $(\overset{\frown}{ZY})$

$\therefore m(\angle ZYE) = m(\angle XZY) = 70°$ (2)

From (1) and (2) we get that:

$m(\angle ZYE) = m(\angle ZEY)$

(b) $\overrightarrow{AD}$ is a tangent:

$\therefore \angle DAC$ is an angle of tangency subtended by $(\overset{\frown}{ABC})$

and $m(\angle DAC) = \frac{1}{2} m(\overset{\frown}{ABC}) = 130°$

And $m(\overset{\frown}{ABC}) = 2 \times 130° = 260°$

$\because m(\overset{\frown}{AC}) + m(\overset{\frown}{ABC}) = 360°$

$\therefore m(\overset{\frown}{AC}) = 360° - 260° = 100°$

$\because \angle B$ is an inscribed angle subtended by $(\overset{\frown}{AC})$

$\therefore m(\angle B) = \frac{1}{2} m(\overset{\frown}{AC}) = 50°$

3

(a) $\because \overleftrightarrow{AD}$ is a tangent and, $\overline{AB}$ is the chord of tangency.

$\therefore m(\angle DAB) = m(\angle C)$ (1) an inscribed angle and a central angle subtended by the same arc $(\overset{\frown}{AB})$

$\because \overline{XY} // \overline{BC}$, $\overline{AC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle C)$ corresponding angle (2)

From (1) and (2) and we get: $m(\angle DAB) = m(\angle AYX)$

$\because m(\angle DAX) = m(\angle AYX)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A, X and Y

(b): Proof:

$\because$ $\overline{XA}$ and $\overline{XB}$ are two tangent segments.

$\therefore$ XA = XB

In Δ XAB:

$\because$ m ($\angle$ XAB) = m ($\angle$ XBA), m ($\angle$ X) = 70°

$\therefore$ m ($\angle$ XAB) = $\frac{180° - 70°}{2}$ = 55° (1)

$\because$ ABCD is a cyclic quadrilateral, m ($\angle$ C) = 125°

$\therefore$ m ($\angle$ DAB) = 180° – 125° = 55° (2)

From (1) and (2)

$\because$ m ($\angle$ XAB) = m ($\angle$ DAB) = 55°

$\therefore$ $\overline{AB}$ bisects $\angle$ DAX

$\because$ m ($\angle$ XBA) = m ($\angle$ DAB) = 55° alternate angles

$\therefore$ $\overline{AD}$ // $\overline{XB}$

4 **(a)** $\because$ MA = MB (radii)

$\therefore$ m ($\angle$ MAB) = m ($\angle$ MBA) = 50°

$\therefore$ m ($\angle$ AMB) = 180° – (50° + 50°) = 80°

$\therefore$ m ($\angle$ BCA) = 40° an inscribed angle and a central angle subtended by the arc ($\overset{\frown}{AB}$)

(b) $\because$ m ($\angle$ E) = 30° , m ($\overset{\frown}{AC}$) = 80°

$\therefore$ 30° = $\frac{180° - m(\overset{\frown}{BD})}{2}$

$\therefore$ m ($\overset{\frown}{BD}$) = 20°

5 **(a)** $\because$ The inscribed circle of the triangle ABC touches its sides at X , Y and Z

$\therefore$ AX = AZ = 3 cm , BX = BY = 4cm , CZ = CY

$\therefore$ CZ = 8 – 3 = 5 cm = CY

$\therefore$ BC = 4 + 5 = 9 cm

(b) $\because$ E is the midpoint of $\overline{XY}$

$\therefore$ $\overline{ME} \perp \overline{XY}$

$\because$ $\overline{AB}$ is a common chord of circles M , N

$\therefore$ $\overline{AB} \perp \overline{MN}$

$\because$ m ($\angle$ EMN) = 130°

$\therefore$ m ($\angle$ C) = 360 – (90 +90 + 130) = 50°